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Evaluation of Reliability Parameters of a Double Unit Repairable System with Preventive Maintenance under Warranty

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Abstract- This paper presents Evaluation of Reliability Parameters of a Double Unit Repairable System with Preventive Maintenance under Warranty. If the Unit under goes PM and works as new after PM. There is a single repairman who always remains with the system. The failure time of the system follows negative exponential distribution while PM and repair time distributions are taken as arbitrary. Supplementary variable technique is adopted to derive the expressions for Reliability, Mean time to system failure and Availability. To highlight the behaviour of Reliability numerical results are considered for particular values of various parameters.

Index Terms: Availability, Mean time to system failure, Reliability, , Supplementary variable Technique, Preventive Maintenance.

1 INTRODUCTION

Many researchers including Shakuntala[1] Kadyan and Promila [3] have analysed the systems under an assumption that the equipment works continuously till failure without considering the preventive maintenance. The continued operation of an equipment will never be perfectly reliable. The equipment is likely to fail during its operation.Therefore, preventive maintenance become an important consideration in the long term performance of the equipment. Maintenance is one of the effective ways of increasing the reliability of the system. Maintenance is considered to be beneficial if the cost of maintenance in terms of money spent and time, is comparably lowwhen compared to cost of the repair after failure of the equipment. For maintained systems, availability is a worth considering measure of performance. This integrates oth reliability parameters and maintainability parameters. Thus it expresses the proportion of down time of the equipment and also warranty acts as an insurance in the situation of early failure of the equipment. In view of these observations in this paper we considered double unit repairable system with preventive maintenance under warranty. There is a single repairman who always with the system. Supplementary variable technique is used to derive the expressions for finding reliability, M.T.S.F and availability. Numerical values are also considered for particular values of various parameters and repair cost.

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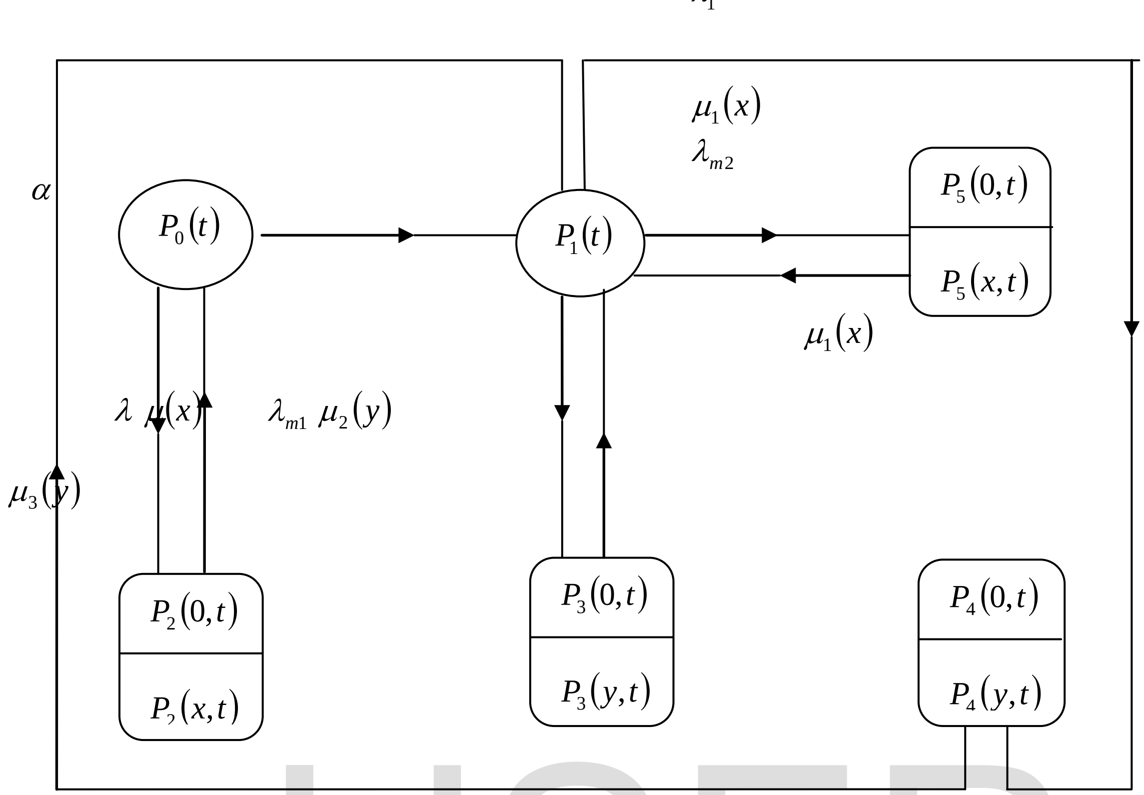
2. SYSTEM TRANSITION STATE DIAGRAM:



3 MODEL ASSUMPTIONS

1. The system has a Double unit Preventive Maintenance under warranty.
2. The repair of the unit within warranty is born by the Manufacturer.
3. Warranty does not apply to product failure due to user induced damage within warranty period.
4. Preventive Maintenance is made during Warranty period.
5. The unit works as new after repair and Preventive Maintenance.
6. The distribution of failure time is taken as negative exponential while the Preventive Maintenance and repair time are considered as arbitrary.
7. Switching is perfect.

4. GENERAL MODEL

4.1 State Specifications

The following states of the system are

S_0 / S_1 The unit is operative within/beyond warranty.

S_2 / S_5 The unit is failed state within/ beyondwarranty.

S_3/ S_4 The units are under preventive maintenance beyond warranty.

4.2 Notations

λ / λ_1 Constant failure rates of units P_2 and P_5.

$\lambda_{m1}, \lambda_{m2}$ Transition rate which the units goes under PM for improvement.

α Transition rate with which warranty of the system is completed.

$\mu(x), S(x)$ Repair rate of the unit and probability density function for theelapsed repair time x within warranty.

$\mu_1(x), S_1(x)$ Repair rate of the unit and probability density function for the elapsed repair time x beyond warranty.

$\mu_2(y), S_2(y)$ PM rate of the first unit and probability density function, for the elapsed PM time y.

$\mu_3(y), S_3(y)$ PM rate of the second unit and probability density function, for the elapsed PM time y.

$P_0(t)/P_1(t)$ Probability density that at time t, when the system is in good state.

$P_i(x,t)$ Probability density that at time t, the system is in state S_i, where i= 2,5 and the system is under repair with elapsed repair time x.

$P_i(y,t)$ Probability density that at time t, the system is in state S_i, where i= 3,4 and the units are under PM with elapsed PM time y.

$P(s)$ Laplace transform of function $P(t)$.

$$S(x) = \mu(x)\exp\left(-\int_0^x \mu(x)dx\right)$$

$$S_1(x) = \mu_1(x)\exp\left(-\int_0^x \mu_1(x)dx\right)$$

$$S_2(y) = \mu_2(y)\exp\left(-\int_0^y \mu(y)dy\right)$$

$$S_3(y) = \mu_3(y)\exp\left(-\int_0^y \mu_3(y)dy\right)$$

5 FORMULATION OF MATHEMATICAL MODEL

Using the Supplementary variable method, the system of differential equations and boundary conditions associated with the model are

$$\left[\frac{d}{dt} + \lambda + \alpha\right]P_0(t) = \int_0^\infty \mu(x)P_2(x,t)dx \quad (1)$$

$$\left[\frac{d}{dt} + \lambda_1 + \lambda_{m1} + \lambda_{m2}\right]P_1(t) = \alpha P_0(t) + \int_0^\infty \mu_2(y)P_3(y,t)dy + \int_0^\infty \mu_3(y)P_4(y,t)dy + \int_0^\infty \mu_1(x)P_5(x,t)dx \quad (2)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu(x)\right]P_2(x,t) = 0 \quad (3)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_2(y)\right]P_3(y,t) = 0 \quad (4)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_3(y)\right]P_4(y,t) = 0 \quad (5)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_1(x)\right]P_5(x,t) = 0 \quad (6)$$

Boundary Conditions:

$$P_2(0,t) = \lambda\ P_0(t) \quad (7)$$

$$P_3(0,t) = \lambda_{m1}\ P_1(t) \quad (8)$$

$$P_4(0,t) = \lambda_{m2}\ P_1(t) \quad (9)$$

$$P_5(0,t) = \lambda_1\ P_1(t) \quad (10)$$

Initial Conditions:

$P_i(0) = 1$ When $i = 0$

$P_i(0) = 0$ When $i \neq 0$ (11)

6 SOLUTION OF THE MODEL

Taking Laplace transforms of equations (1) - (10) and using (11) we obtain

$$[s + \lambda + \alpha]P_0(s) = 1 + \int_0^\infty \mu(x)P_2(x,s)dx \quad (12)$$

$$[s+\lambda_1+\lambda_{m1}+\lambda_{m2}]P_1(s)= \alpha P_0(s)$$
$$+\int_0^\infty \mu_2(y)P_3(y,s)dy+\int_0^\infty \mu_3(y)P_4(y,s)dy \quad (13)$$
$$+\int_0^\infty \mu_1(x)P_5(x,s)dx$$

$$\left[s+\frac{\partial}{\partial x}+\mu(x)\right]P_2(x,s)=0 \quad (14)$$

$$\left[s+\frac{\partial}{\partial y}+\mu_2(y)\right]P_3(y,s)=0 \quad (15)$$

$$\left[s+\frac{\partial}{\partial y}+\mu_3(y)\right]P_4(y,s)=0 \quad (16)$$

$$\left[s+\frac{\partial}{\partial x}+\mu_1(x)\right]P_5(x,s)=0 \quad (17)$$

$$P_2(0,s) = \lambda\ P_0(s) \quad (18)$$

$$P_3(0,s) = \lambda_{m1}\ P_1(s) \quad (19)$$

$$P_4(0,s) = \lambda_{m2}\ P_1(s) \quad (20)$$

$$P_5(0,s) = \lambda_1\ P_1(s) \quad (21)$$

Taking Integration of equations14),(15),(16), (17), we get the following equations:

$$P_2(x,s)=\lambda P_0(s)\exp\left(-sx-\int_0^x \mu(x)dx\right) \quad (22)$$

$$P_3(y,s)=\lambda_{m1}P_1(s)\exp\left(-sy-\int_0^y \mu_2(y)dy\right) \quad (23)$$

$$P_4(y,s)=\lambda_{m2}P_1(s)\exp\left(-sy-\int_0^y \mu_3(y)dy\right) \quad (24)$$

$$P_5(y,s)=\lambda_1 P_1(s)\exp\left(-sx-\int_0^x \mu_1(x)dx\right) \quad (25)$$

Using equations (18) and(22) in (12) then we get

$$P_0(s)=\frac{1}{A(s)} \quad (26)$$

where $A(s)=s+\alpha+\lambda(1-S(s))$ (27)

Using equations (23),(24) and (25) in (13) then we get

$$[s+\lambda_1+\lambda_{m1}+\lambda_{m2}]P_1(s) = \alpha P_0(s)$$
$$+\int_0^\infty \mu_2(y)\lambda_{m1}P_1(s)\exp\left(-sy-\int_0^y \mu_2(y)dy\right)dy$$
$$+\int_0^\infty \mu_3(y)\lambda_{m2}P_1(s)\exp\left(-sy-\int_0^y \mu_3(y)dy\right)dy$$
$$+\int_0^\infty \mu_1(x)\lambda_1 P_1(s)\exp\left(-sx-\int_0^x \mu_1(x)dx\right)dx$$

$$= \frac{\alpha}{A(s)}+\lambda_{m1}P_2(s)S_2(s)+\lambda_{m2}P_1(s)S_3(s)+\lambda_1 P_1(s)S_1(s)$$

$$P_1(s)=\frac{B(s)}{A(s)} \quad (28)$$

Where $B(s)=\dfrac{\alpha}{s+\lambda_1(1-S_1(s))+\lambda_{m1}(1-S_2(s))+\lambda_{m2}(1-S_3(s))}$ (29)

Now, the Laplace transform of the probability that the system is in the failed state is given by

$$P_2(s)=\int_0^\infty P_2(x,s)dx = \frac{\lambda C(s)}{A(s)} \quad (30)$$

where $C(s)=\dfrac{1-S(s)}{s}$ (31)

$$P_3(s)=\int_0^\infty P_3(y,s)dy = \frac{\lambda_{m1}B(s)D(s)}{A(s)} \quad (32)$$

where $D(s)=\dfrac{1-S_2(s)}{s}$ (33)

$$P_4(s)=\int_0^\infty P_4(y,s)dy = \frac{\lambda_{m2}B(s)E(s)}{A(s)} \quad (34)$$

where $E(s)=\dfrac{1-S_3(s)}{s}$ (35)

$$P_5(s)=\int_0^\infty P_5(y,s)dy = \frac{\lambda_1 B(s)F(s)}{A(s)} \quad (36)$$

where $F(s)=\dfrac{1-S_1(s)}{s}$ (37)

We can easily verify that

$$P_0(s)+P_1(s)+P_2(s)+P_3(s)+P_4(s)+P_5(s)=\frac{1}{s}$$
(38)

7 EVALUATION OF LAPLACE TRANSFORMS UP AND DOWN STATE PROBABILITIES

The Laplace transforms of the probabilities that the system is in Up State ($P_{up}(t)$)(i.e., Good state) and Down State ($P_{down}(t)$)(i.e., failed State) at time t are as follows

$$Av(s) \text{ or } P_{up}(s)=P_0(s)+P_1(s)=\frac{1+B(s)}{A(s)} \quad (39)$$

$$P_{down}(s)=P_2(s)+P_3(s)+P_4(s)+P_5(s)$$

$$P_{down}(s)=\frac{\lambda C(s)+\lambda_{m1}B(s)D(s)+\lambda_{m2}B(s)E(s)+\lambda_1 B(s)F(s)}{A(s)}$$
(40)

8 STEADY STATE BEHAVIOUR OF THE SYSTEM

Using Abel's Lemma i.e., $\underset{t\to\infty}{Lt}\, F(t)=\underset{s\to 0}{Lt}\, sF(s)=F$ in equations (39) and (40), Provided the limit on the right hand side exists, the following time independent probabilities have been obtained:

$$P_{up}=\frac{1}{1-\lambda_1 S_1^1(0)-\lambda_{m1}S_2^1(0)-\lambda_{m2}S_3^1(0)} \quad (41)$$

$$P_{down}=\frac{-\lambda_1 S_1^1(0)-\lambda_{m1}S_2^1(0)-\lambda_{m2}S_3^1(0)}{1-\lambda_1 S_1^1(0)-\lambda_{m1}S_2^1(0)-\lambda_{m2}S_3^1(0)} \quad (42)$$

9 RELIABILITY OF THE SYSTEM R(t):

The differential equations for reliability of the system are:

$$\left[\frac{d}{dt}+\lambda+\alpha\right]P_0(t)=0 \quad (43)$$

$$\left[\frac{d}{dt}+\lambda_1+\lambda_{m1}+\lambda_{m2}\right]P_1(t)=\alpha P_0(t) \quad (44)$$

Taking Laplace transform of equations (43) and (44), using (11),we get:

$$[s+\lambda+\alpha]P_0(s)=1 \quad (45)$$

$$[s+\lambda_1+\lambda_{m1}+\lambda_{m2}]P_1(s)=\alpha P_0(s) \quad (46)$$

The solution can be written as:

$$P_0(s)=\frac{1}{(s+\alpha+\lambda)} \quad (47)$$

$$P_1(s)=\frac{\alpha}{(s+\alpha+\lambda)(s+\lambda_1+\lambda_{m1}+\lambda_{m2})} \quad (48)$$

$$R(s)=P_0(s)+P_1(s) \quad (49)$$

$$=\frac{1}{s+\lambda+\alpha}+\frac{\alpha}{(s+\lambda+\alpha)(s+\lambda_1+\lambda_{m1}+\lambda_{m2})}$$

Taking inverse Laplace transform, we get

$$R(t)=e^{-(\lambda+\alpha)t}\left[\frac{\lambda-\lambda_1-\lambda_{m1}-\lambda_{m2}}{\lambda+\alpha-\lambda_1-\lambda_{m1}-\lambda_{m2}}\right]+e^{-(\lambda_1+\lambda_{m1}+\lambda_{m2})t}\left[\frac{\alpha}{\lambda+\alpha-\lambda_1-\lambda_{m1}-\lambda_{m2}}\right] \quad (50)$$

10 MEAN TIME TO SYSTEM FAILURE (MTSF):

$$MTSF=\int_0^\infty R(t)dt$$

$$=\frac{\lambda-\lambda_1-\lambda_{m1}-\lambda_{m2}}{(\lambda+\alpha)(\lambda+\alpha-\lambda_{m1}-\lambda_{m2})}+\frac{\alpha}{(\lambda+\alpha-\lambda_1-\lambda_{m1}-\lambda_{m2})(\lambda_1+\lambda_{m1}+\lambda_{m2})}$$
(51)

11 NUMERICAL ANALYSIS

Table:1

$\lambda_1=0.02,\ \lambda_{m1}=0.04,\ \lambda_{m2}=0.06,\ \alpha=0.005$

Time in days	Reliability		
	$\lambda=0.01$	$\lambda=0.02$	$\lambda=0.03$
1	0.9898	0.98	0.9699
2	0.9792	0.9598	0.941
3	0.9719	0.9398	0.9122
4	0.9572	0.9199	0.8841
5	0.9458	0.9001	0.85666

Table:2

$\lambda=0.02,\ \lambda_{m1}=0.04,\ \lambda_{m2}=0.06,\ \alpha=0.005$

Time in days	Reliability		
	$\lambda_1=0.01$	$\lambda_1=0.02$	$\lambda_1=0.03$
1	0.98	0.98	0.9799
2	0.9599	0.9598	0.9598
3	0.9401	0.9398	0.9397
4	0.9202	0.9199	0.9196
5	0.9005	0.9001	0.8997

Table:3

$\lambda=0.02,\ \lambda_{m1}=0.04,\ \lambda_1=0.03,\ \alpha=0.005$

Time in days	Reliability		
	$\lambda_{m2} = 0.01$	$\lambda_{m2} = 0.02$	$\lambda_{m2} = 0.03$
1	0.98	0.98	0.98
2	0.9607	0.9605	0.9601
3	0.9405	0.9402	0.9401
4	0.921	0.9205	0.9202
5	0.9018	0.9011	0.9005

Table:4

$\lambda = 0.02, \lambda_{m1} = 0.04, \lambda_{m2} = 0.06, \alpha = 0.005$

Time in days	Reliability		
	$\lambda_{m1} = 0.01$	$\lambda_{m1} = 0.02$	$\lambda_{m1} = 0.03$
1	0.98	0.98	0.9799
2	0.9599	0.9598	0.9598
3	0.9401	0.9398	0.9397
4	0.9202	0.9199	0.9196
5	0.9005	0.9001	0.8997

Table:5

$\lambda = 0.02, \lambda_{m1} = 0.04, \lambda_1 = 0.03, \lambda_{m2} = 0.06$

Time in days	Reliability		
	$\alpha = 0.003$	$\alpha = 0.005$	$\alpha = 0.007$
1	0.98	0.9799	0.9799
2	0.9602	0.9598	0.9594
3	0.9405	0.9397	0.9388
4	0.921	0.9196	0.9182
5	0.9017	0.8997	0.8976

12 INTERPRETATION OF THE RESULTS Tables1,2,3,4 and 5 show the behaviour of system reliability. Tables 1,2,3 and4 indicate that the reliability of the system decreases with the increase of failure rates λ, λ_1 and transition rates $\lambda_{m1}, \lambda_{m2}$ with respect to time and fixed values of other parameters. From table 5 it is analysed that the reliability of the system increases with the decrease of rate of completion of warranty α with respect to time .

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